Taken from the Learning Progression: K, Counting and Cardinality; K–5, Operations and Algebraic Thinking

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| **Methods used for solving single-digit addition and subtraction problems**  Level 1. Direct Modeling by Counting All or Taking Away.  Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.  Level 2. Counting On.  Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count. For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).  Level 3. Convert to an Easier Problem.  Decompose an addend and compose a part with another addend.  See Appendix in the learning progression for examples and further details. |

Counting on should be seen as a thinking strategy, not a rote method. It involves seeing the first addend as embedded in the total, and it involves a conceptual interplay between counting and the cardinality in the first addend (shifting from the cardinal meaning of the first addend to the counting meaning). Finally, there is a level of abstraction involved in counting on, because students are counting the words rather than objects. Number words have become objects to students.

Level 3 methods involve decomposing an addend and composing it with the other addend to form an equivalent but easier problem. This relies on properties of operations. Students do not necessarily have to justify their representations or solution using properties, but they can begin to learn to recognize these properties in action and discuss their use after solving.

There are a variety of methods to change to an easier problem. These draw on addition of three whole numbers.1.OA.2 A known addition or subtraction can be used to solve a related addition or subtraction by decomposing one addend and composing it with the other addend. For example, a student can change 8 + 6 to the easier 10 + 4 by decomposing 6 = 2 + 4 and composing the 2 with the 8 to make 10: 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14.

This method can also be used to subtract by finding an unknown addend: 14 - 8 = 🞏, so 8 + 🞏 = 14, so 14 = 8 + 2 + 4 = 8+ 6, that is 14 = 8 + 6. Students can think as for adding above (stopping when they reach 14), or they can think of taking 8 from 10, leaving 2 with the 4, which makes 6. One can also decompose with respect to ten: 13 - 4 = 13 – 3 - 1 = 10 - 1 = 9, but this can be more difficult than the forward methods.

These make-a-ten methodshave three prerequisites reaching back to Kindergarten:

a. knowing the partner that makes 10 for any number (K.OA.4 sets the stage for this),

b. knowing all decompositions for any number below 10 (K.OA.3 sets the stage for this), and

c. knowing all teen numbers as 10 + *n* (e.g., 12 = 10 + 2, 15 = 10 + 5, see K.NBT.1 and 1.NBT.2b).

The word *fluent* is used in the Standards to mean “fast and accurate.” Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., “adding 0 yields the same number”), and knowing some answers from the

use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students. The extensive work relating addition and subtraction means that subtraction can frequently be solved by thinking of the related addition, especially for smaller numbers. It is also important that these patterns, strategies and decompositions still be available in Grade 3 for use in multiplying and dividing and in distinguishing adding and subtracting from multiplying and dividing. So the important press toward fluency should also allow students to fall back on earlier strategies when needed. By the end of the K–2 grade span, students have sufficient experience with addition and subtraction to know single-digit sums from memory;2.OA.2 as should be clear from the foregoing, this is not a matter of instilling facts divorced from their meanings, but rather as an outcome of a multi-year process that heavily involves the interplay of practice and reasoning.